OPTIMISATION OF SHIP HULLS CONSIDERED AS SLENDER-BODIES

Optimisation de carènes dans l'approximation de corps élancé

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ABSTRACT

The slender body approximation, first introduced by [Rankine, 1886] for the study of a potential flow around a thin obstacle in an infinite domain states that a thin obstacle has the same effect on the flow as a distribution of source/well doublets on a line. This model has been extended later for linear free surface flows and lead to the explicit formulas of [Michell, 1898] and [Sretenski, 1937] for the computation of the wave making resistance of a slender ship moving with an uniform velocity. These formulas provide the wave making resistance as a quadratic function of the hull "offset" function. Although very well known and widely used for ships with polynomial shapes (such as the Wigley hull), very few attempts of hull optimisation have been made. Some existing results suggesting that these optimisation problems are in fact singular (non-existence or non-uniqueness of optimal shapes). A regularisation of the problem limiting the surface area (and hence the frictional resistance) will be presented along with numerical results we have established in [Dambrine, Pierre, Rousseaux, 2015] in the case of infinite depth and width. In the aforementioned article, the hull offset function is supposed have a fixed compact support (the support actually represents the longitudinal cut of the hull). Setting a constant support is a quite restrictive condition. We will show how to partly remove this condition by a geometric optimisation method in the spirit of [Allaire, 2007].

KEY WORDS

Optimisation, Shape optimisation, Slender bodies, Michell, Sretenski, Mathematics.

RESUME

Optimisation de carènes dans l'approximation de carènes élancées
L'approximation de corps élancés, introduite dans [Rankine, 1886] pour l'étude d'écoulements potentiels autour d'un obstacle fin dans un milieu infini consiste à assimiler un tel obstacle à une distribution de sources/puits le long d'une ligne de symétrie. Ce modèle a été étendu plus tard aux écoulement à surface libre linéaires et conduit aux formules analytiques de [Michell, 1898] et [Sretenski, 1937] pour le calcul de la résistance de vagues subie par une carène élancée avançant avec une vitesse constante. Ces formules donnent la résistance à l'avancement comme une fonction quadratique de la fonction "offset" de la carène, qui joue ici le rôle de paramètre de forme. Bien qu'elle soit bien connue pour des carènes de formes polynomiale (comme la carène de Wigley), peu de travaux abordent l'optimisation des formes de carènes dans un contexte plus général. Quelques résultats existants suggèrent que ce problème est singulier (non-existence ou non unicité de carènes optimales). Un terme de régularisation limitant la surface de la coque (et ainsi la résistance frictionnelle) sera introduit permettant d'établir des résultats d'existence d'unicité et de régularité du problème d'optimisation. Quelques résultats numériques établis dans [Dambrine, Pierre, Rousseaux, 2015] seront présentés. Dans l'article mentionné précédemment, la fonction "offset" de la carène est supposée avoir un support compact et fixe (le support représente la section longitudinale de la carène), ce qui représente une condition plutôt restrictive, que nous tenterons partiellement de lever par le biais de méthodes d'optimisation géométrique similaires à celles développées dans [Allaire, 2007].

MOTS-CLEFS

Optimisation, Optimisation de formes, Corps élancés, Michell, Sretenski, Mathématiques.

1. INTRODUCTION

Although it is only one component in the long chain of efficiency of a ship, the minimisation of the wave-making resistance is interesting for both the reduction of fuel consumption and the reduction of the impact of wash waves on the banks of rivers. This wave making resistance depend on the dispersive context (finite or infinite depth/width, stratification,…), the ship's velocity and also the shape of the ship.

In this article we present some results of ship's hull shape optimisation in a simplified physical context (slender-ship models). This problem has been studied theoretically in [Kostyukov, 1968], however, few numerical studies have been carried out on general shapes. Our goal is to remove as many geometrical constraints as possible on the space of shapes as we progress. We will start with optimisation of ship hull with a given longitudinal section (we call the support of the hull), and move on to actual shape optimisation of the support. In the following section we describe a simplified representation of the ship's hull with an offset function.

1.1 Representation of the ship's hull

Let us consider a ship moving at a constant speed, along the x axis. We will call y the transverse axis, and z the depth axis. The "centerplane" of the ship is defined by the (x,z) plane. Without loss of too much generality, we will suppose that the hull is symmetric with respect to the centerplane. In the following, we will consider that the surface of the hull is defined by the graph of a positive function f which, for every point of the centerplane returns the half-width of the hull (see figure 1).

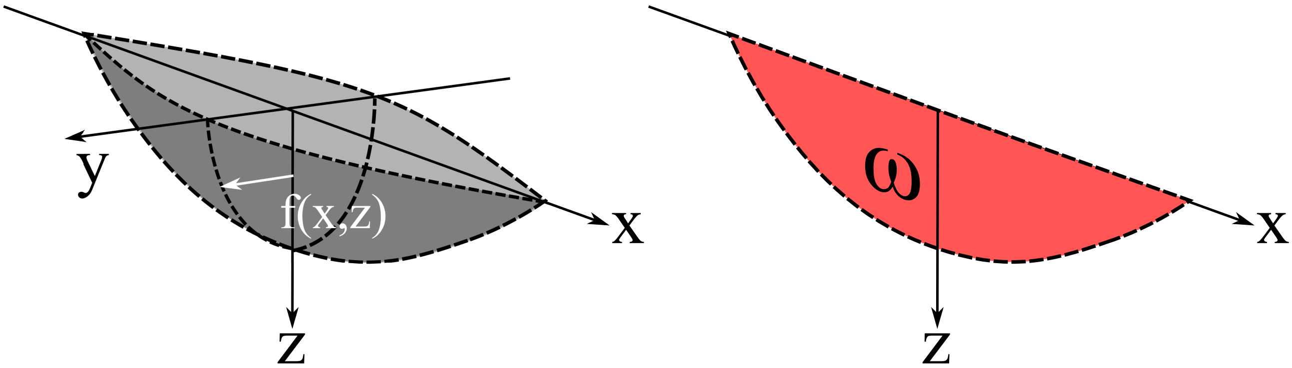

Figure 1: representation of the ship's hull with an offset function. Right: definition of the offset function f ; left: representation of the support ω of the function.

We will call the "support" of the hull the closure of the set of points (x,z) on which $f(x,z) \neq 0$. This support will be often called ω. In the following paper, in order to avoid infinite length and draught ships, we will consider ω to be bounded.

The "slender-ship hypothesis" which is required for the simplified models described in the next section states that the gradient of f has to be uniformly small on the centerplane. In order to avoid jumps (infinite gradients), we will assume that f "touches" zero on the border of ω. The following section details the simplified analytic model used in our optimisation problem, which makes use of the aforementioned "slender-ship hypothesis".

1.2 Wave-making resistance formula

In the following study we will use a simplified model to describe the wave making resistance of a ship which is based on Michell's formula [Michell, 1898]:

$$R_w = \frac{4\rho g \nu^3}{\pi} \int_1^{\infty} |I(\lambda, f)|^2 \frac{\lambda^4}{\sqrt{\lambda^2 - 1}} \mathrm{d}\lambda \,, \tag{1}$$

with:

$$I(\lambda, f) = \int_{\mathbb{R}^2} f(x,z)\, e^{-\nu\lambda^2 z}\, e^{i\nu\lambda x}\, \mathrm{d}x\mathrm{d}z \,, \tag{2}$$

where $\nu = g/U^2$, with U being the ship's velocity. This formula can be obtained by solving analytically the Neumann-Kelvin model (see [Kuznetsov, 2004] for a detailed study of the model) for ship waves, which is entirely based on the following assumption: the wake waves are linear with respect to the hull's offset function. This assumption implies that (1)-(2) is valid in the limit of slender/thin ships.

It is worth noticing that Michell's formula can be rewritten in the following kernel-based form which is more appropriate for theoretical studies and optimisation:

$$R_w = \int_{\mathbb{R}^2}\int_{\mathbb{R}^2} K(x,z,x',z') f(x,z) f(x',z')\,\mathrm{d}x\mathrm{d}z\mathrm{d}x'\mathrm{d}z'\,, \tag{3}$$

where the kernel K (that we call Michell's kernel) is given by:

$$K(x,y,x',z') = \frac{4\rho g \nu^3}{\pi}\int_1^\infty \frac{\lambda^4}{\sqrt{\lambda^2-1}} e^{-\nu(\lambda^2(z+z')+i\lambda(x-x'))}\mathrm{d}\lambda\,. \tag{4}$$

With this formulation, we can see easily that the wave-making resistance is a quadratic function of the ship's offset function. The problem of finding the offset function that minimises the wave-making resistance is hence a problem of quadratic optimisation for which specific efficient numerical techniques can be applied.

Although our paper focuses on Michell's kernel to describe wave-making resistance in unrestricted waters, the same method can be applied to various physical situations, with the same kernel-based formulation as in (3), but with different kernel functions. Here are some examples of such formula:

- Sretensky's formula for finite depth waters (see [Sretensky, 1937]),
- Sretensky's formula for finite width channels with infinite depth (see [Stretensky, 1936]),
- Keldysh-Sedov's formula for both finite width and depth (see [Keldysh, Sedov, 1937]).

1.3 Theoretical issues : the need for regularisation

Let us consider the following problem:

$$f^* = \arg\min_{f\in K} R_w(f) \tag{5}$$

where K represents the set of constraints on the hull. First we have already stated that f should be positive. In order to avoid the trivial zero solution, we need to impose a volume for the hull :

$$\int_\omega f(x,z)\,\mathrm{d}x\mathrm{d}z = \frac{V}{2}\,, \tag{6}$$

where V is the volume of the hull. Moreover, as stated before, the restriction of f on the border of ω should be zero. Since we need to be able to define the gradient of f, its natural class of regularity should be $C^1(\omega)$. Summing up all the above constraints, K writes:

$$K = \left\{ f\in C^1(\omega) : f|_{\partial\omega} = 0\,,\ f\geq 0\,,\ \int_\omega f = V/2 \right\}\,. \tag{7}$$

Various theoretical results developed in [Kostyukov, 1968] suggest that the problem is singular. The main idea is to build zero wave-making resistance hulls : if one can find $f^*\in K$ such that $R_w(f^*)=0$, then for any scalar value λ, we have : $R_w(\lambda f^*)=0$, hence the minimiser is not unique (there is an infinity of minima). In [Kostyukov, 1968] zero resistance hulls are found in the case of non-bounded supports, and, in the case of bounded support, for non-positive f. Further numerical tests in [Dambrine, Pierre, Rousseaux, 2015] seem to confirm the fact that the problem of minimizing the wave-making resistance is ill-posed.

A solution to this problem is to add a regularisation term which prevents the solution to become too oscillatory, leading to large surface area hulls. The new problem writes:

$$f^* = \arg\min_{f\in K}\left\{ \epsilon\int_\omega |\nabla f|^2 + R_w(f) \right\} \tag{8}$$

where ϵ is a (typically small) regularisation parameter. Note that this term can be related to a frictional resistance term. A simplified model of frictional resistance writes:

$$R_v = \frac{1}{2}\rho\, C_w U^2 S\,, \tag{9}$$

where C_w is a drag coefficient and S the surface area of the hull. In our case this surface area writes:

$$S = \int_\omega \sqrt{1+|\nabla f|^2}\,. \tag{10}$$

Developping at first order with respect to $|\nabla f|$, we obtain the following approximation:

$$S \sim \int_\omega (1+|\nabla f|^2) = |\omega| + \int_\omega |\nabla f|^2\,. \tag{11}$$

When the support is considered to be fixed, the first term $|\omega|$ is constant, hence the minimisation problem (8) with $\epsilon = \rho C_w U^2$ is actually the minimisation of the sum of the wave making resistance and frictional resistance. In [Dambrine, Pierre, Rousseaux, 2015], we have proven the existence of a unique solution of the regularised optimization problem (8).

3. HULL OPTIMISATION WITH A CONSTANT SUPPORT

Let us recall the regularised optimisation problem (8):

$$f^* = \arg\min_{f\in K} \epsilon \int_\omega |\nabla f(x,z)|^2 \mathrm{d}x\mathrm{d}z + \int_\omega\int_\omega K(x,z,x',z')f(x,z)f(x',z')\mathrm{d}x\mathrm{d}z\mathrm{d}x'\mathrm{d}z' \,. \tag{12}$$

We use a finite-element approach in order to solve this problem. Let us decompose f on a finite element basis $(e_i)_{i=1..N}$:

$$f = \sum_{i=1}^N f_i e_i \,. \tag{13}$$

By injecting the expression (13) into (12), and by denoting $F = (f_i)_{i=1..N}$ the vector of coefficients, the problem becomes the following quadratic programming problem:

$$F^* = \arg\min_{F\in K_N} {}^tFMF \,, \tag{14}$$

where $M = (m_{i,j})_{\substack{i=1..N\\ j=1..N}}$, with :

$$m_{i,j} = \int_\omega \nabla e_i(x,z)\cdot\nabla e_j(x,z)\mathrm{d}x\mathrm{d}z + \int_{\omega^2} K(x,z,x',z')e_i(x,z)e_j(x',z')\mathrm{d}x\mathrm{d}z\mathrm{d}x'\mathrm{d}z' \,, \tag{15}$$

and K_N is the discrete set of affine constraints, which writes in the case of P1 or Q1 finite elements:

$$\begin{cases} f_i \geq 0 \,, \\ f_j = 0 \,, \text{ for the boundary points } j \,, \\ \displaystyle\sum_{i=1}^N \left(\int_\omega e_i\right) f_i = V/2 \,. \end{cases} \tag{16}$$

The matrix detailed in (15) can be computed using standard finite element softwares such as FreeFem. The computation of Michell's kernel is performed by using a quadrature method described in [Tarafder, 2007] which preserves the positivity of the matrix. We used the algorithm of [Uzawa, 1958] in order to solve the quadratic programming problem (14-16).

The following numerical tests show the results of optimisation for a fixed rectangular support $\omega = [-L/2, L/2] \times [0, D]$, with $L = 2m \,,\ D = 0.2m$. Let us recall that Michell's kernel depends on the velocity of the ship, and hence each hull will be optimised for a given velocity. For all the following tests, the ship's velocity will be defined using the length-based Froude number, which is a more relevant quantity to measure the effect of ship waves:

$$U = Fr\,\sqrt{gL} \,. \tag{17}$$

The regularisation parameter is defined according to (9), with a drag coefficient of 0.01 (which is a reasonable value for a streamlined body). The figure 2 shows the result of our algorithm for various values of the targeted Froude number. We obtain very different designs depending on this parameter. The most notable transition is the apparition of a bulbous bow and stern for values of the Froude number for which the wave resistance is usually ramping up very quickly.

In order to understand the efficiency of these bulbous bows, we have compared the profile of wave resistance of two different optimised hulls with a standard shape called the parabolic Wigley hull with the same volume. In figure 3 we notice that the each optimised hull are efficient in the Froude regimes they were designed for. The optimised hull for a Froude number of 0.3 (the canoë-like shape in figure 1) behaves well for low velocities, but is less efficient than the Wigley hull for higher velocities, whereas the optimised hull for a Froude number of 0.5 (the bulbous bow and stern shape in figure 1) is not efficient at all for low Froude numbers.

Bulbous bows are classical in hull design however, bulbous stern are never encountered. This is due to the fact that we have not taken into account propulsion in our modelling and the propeller efficiency yields a lot of geometrical constraints on the stern, which are too complex to be taken into account in this study.

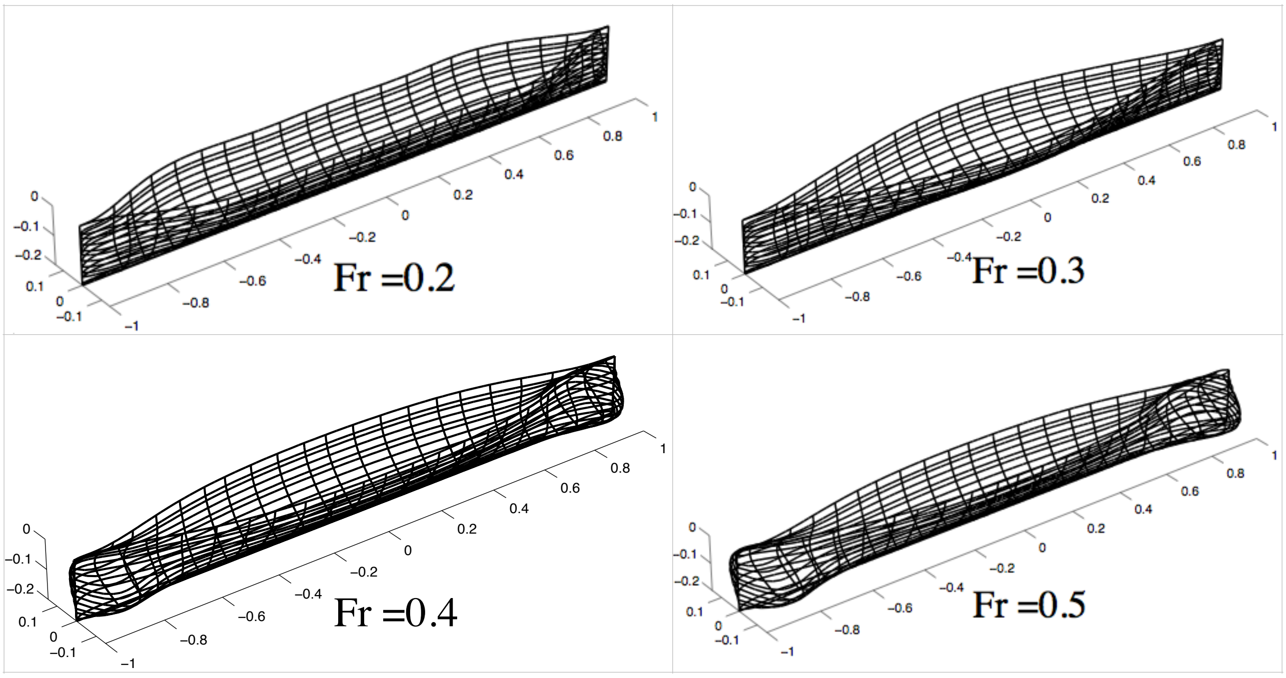

Figure 2: A view of the hulls obtained though our optimisation algorithm for a fixed rectangular support, with various values of the target Froude number.

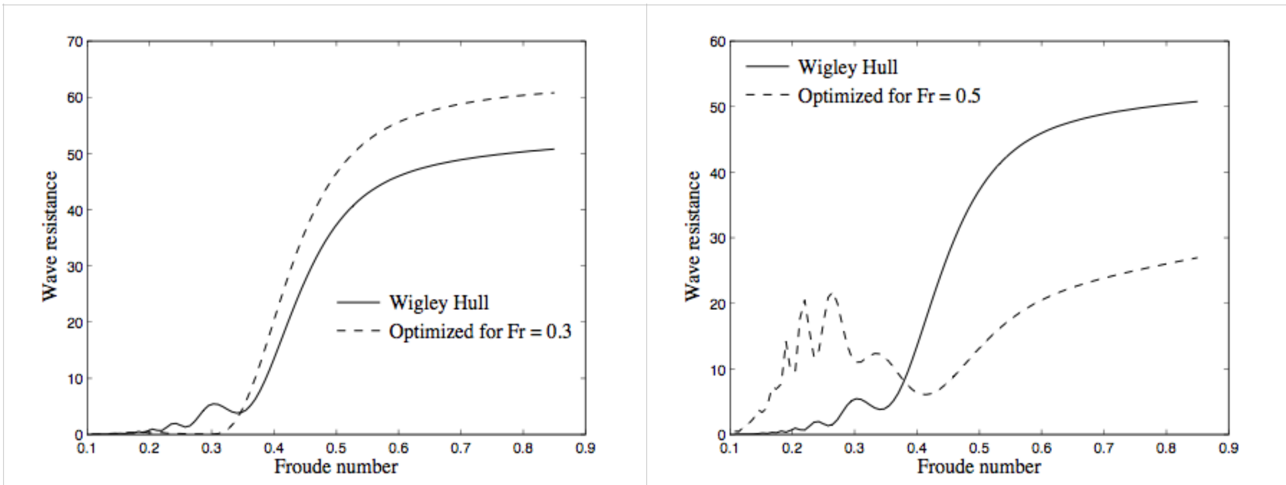

Figure 3: Representations of the wave resistance profiles of two optimised hulls for different Froude numbers (in plain line) compared to an equivalent parabolic Wigley hull (in dashed line).

4. TOWARDS SUPPORT OPTIMISATION

The previous simulations have shown that the efficiency of a hull can be improved by modifying its shape, however, we assumed that the support (*i.e.* the longitudinal section) of the offset function is given. In this section we ask ourselves the question: can we further improve the situation by optimising also the shape of the support on which it is defined ? Such an optimisation problem would write :

$$\omega^* = \arg\min_{\omega} J(\omega)\,, \tag{18}$$

where:

$$J(\omega) = \min_{f\in K}\ \epsilon(|\omega| + \int_{\omega} |\nabla f|^2) + R_w(f)\,. \tag{19}$$

Of course, in (18), ω has to lie in a set of admissible supports that we need to define. One of the biggest issues is that there is no general parametrisation of such supports, and without any structure, such sets can be extremely complex and lead to ill-posed problems for optimisation.

4.1 Geometric optimisation : a brief description

The solution we adopted here, is the approach developed in [Allaire, 2007] which consists in restricting the set of "admissible" supports to the supports that can be reached through a smooth and reversible deformation of a reference support. Hence the parameter that defines a support is this deformation. The advantage of this method is that it provides a good notion of gradient which will be useful to design an optimisation algorithm. However, its main drawback is its lack of generality : for example, it is not possible to add holes in the shape or secondary components to the hull since all the shapes are obtained through the smooth deformation of an initial shape. Note that the relation between $J(\omega)$ and T is very complex and lacks the main ingredient to ensure the uniqueness of a minimum : convexity. Other theoretical tools ca be used to obtain uniqueness but these methods are non-constructive.

A notion of shape gradient can be built on the idea that we can compute the first order Taylor expansion of the functional $J(\omega)$ with respect to a small deformation $T = Id + \theta$, where θ is a (small) velocity field which represents the instantaneous deformation of the support. The shape gradient of $J(\omega)$ provides a small deformation field that acts as a descent direction for our support. By repeating this process until convergence, our gradient descent algorithm provides a (at least) local minimiser for ω.

4.1 Numerical calculations

The computation of the shape gradient is not detailed here, however, we give some numerical results to illustrate the difficulties related to this method. All the computations below have been performed with the help of FreeFem++, in particular, we made an extensive use of the mesh moving and re-meshing methods provided by this software.

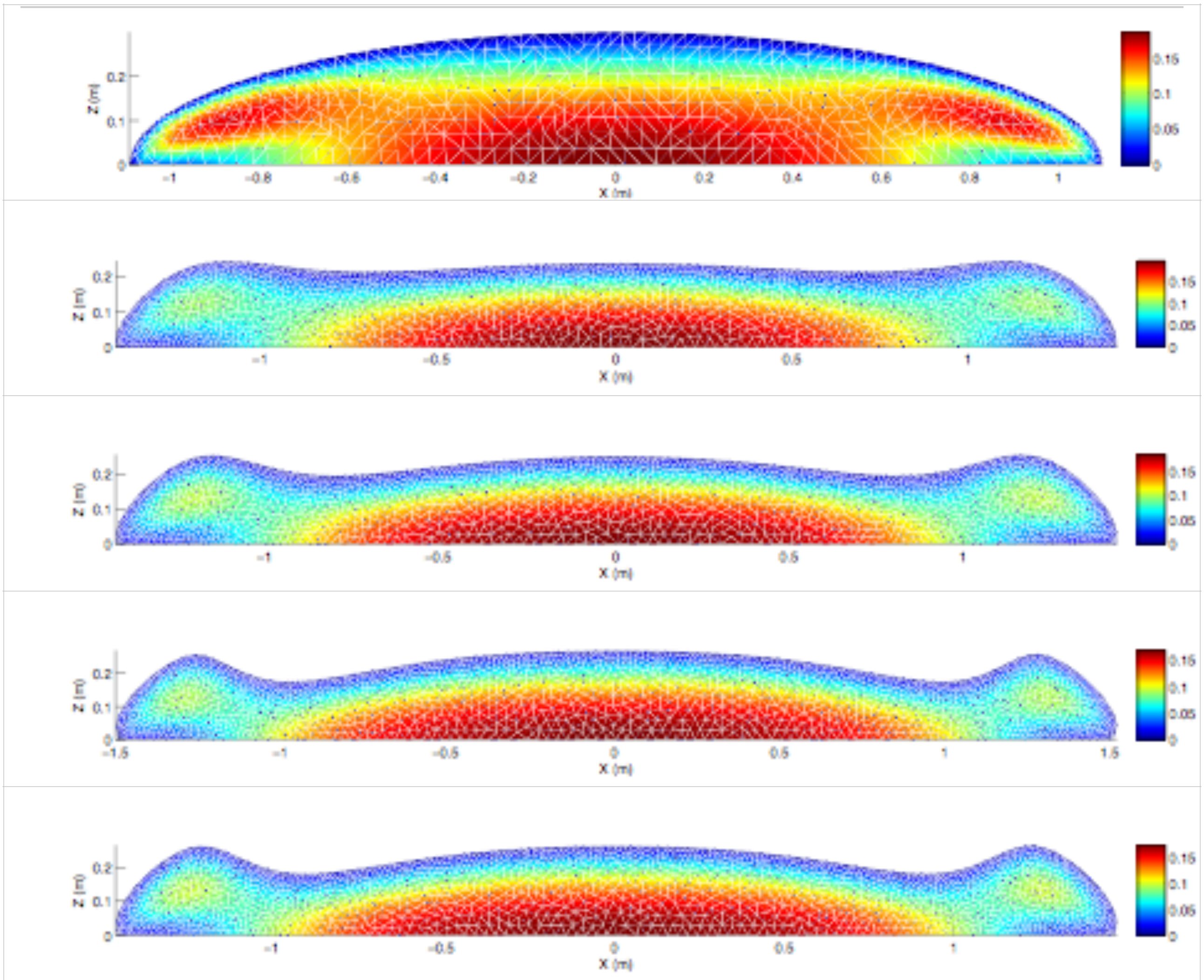

Figure 4: representation of the hull shape for different steps of the support optimisation algorithm with a target Froude number of 0.3. The last step represents the converged solution. The colours represent the optimised offset function values.

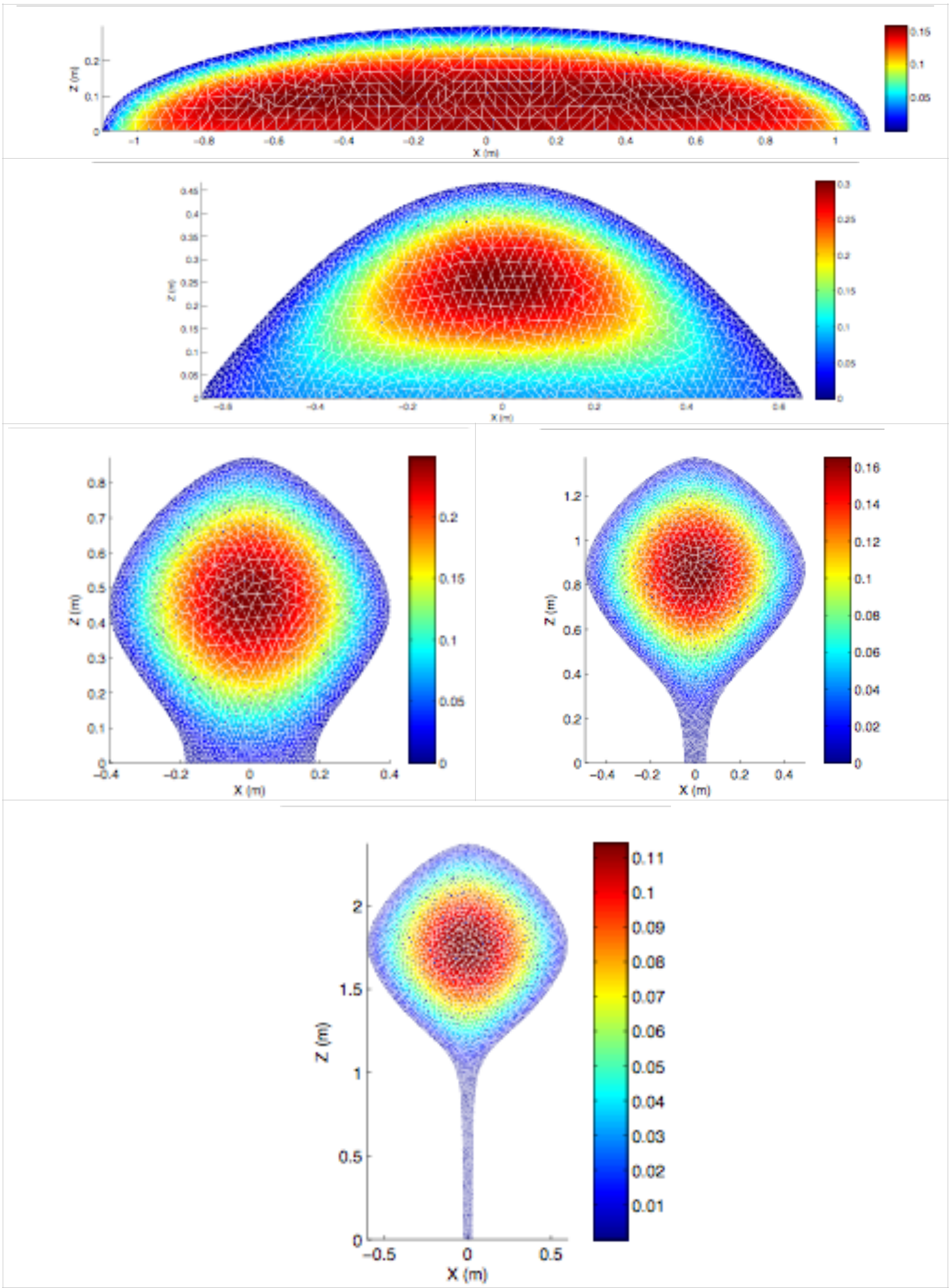

Figure 5: representation of the hull shape for different steps of the support optimisation algorithm with a target Froude number of 0.75. The last step represents the converged solution. The colours represent the optimised offset function values.

In figure 4 and 5, we can see two example of support optimisation based on shape gradient descent. These examples illustrate the sensitivity of this method on the parameters. In the first case, a steady state is reached in finite time, and the local minimum we obtain seems to be a reasonable design for a ship. In the second case the whole volume sinks and no steady-state is reached. This is due to the fact that it is possible to build non-converging minimising sequence of hulls by sending all the mass away from the free surface (thus generating no waves). However the benefit of sending the mass away from the surface has to be counter-balanced with the cost of stretching the support. Recall that no topological changes are allowed, hence the support will always touch the surface, even through a thin filament. In the particular phenomenon depicted on figure 5, the “sinking" effect seems to overcome this cost.

As we got rid of the fixed support constraint, we observed new situations, such as the one depicted in figure 5, which is trivial from the point of view of naval architecture (less so in the point of view of shape optimisation). The addition of new constraints (such as a bounding box or a fixed center of gravity) in the future should allow us to obtain more reasonable designs with the geometrical optimisation process.

5. CONCLUSION

We have conducted a study of the minimisation of the wave-making resistance of a ship by the use of simplified analytic models, and trying to consider shapes with as much generality as possible. First, by setting the support constant we obtain a parametric optimisation problem that lies in the family of quadratic programming problems, which can be solved efficiently, leading to designs featuring bow and stern bulbs. The optimisation of the support is a way to solve the problem with more generality, but some care has to be taken in order to avoid trivial solutions. I future works we will investigate the problem by removing the slender-ship hypothesis. This should lead to even more generality for the possible shapes we may obtain, and of course to new issues for the definition of the right set of constraints to associate to our problem.

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