Comment on "Momentum Transfer from Quantum Vacuum to Magnetoelectric Matter"

In the Letter of van Tiggelen *et al.* [1], one can read "The Galilean transformation imposes that $\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}$ and $\mathbf{B}' = \mathbf{B} - (1/c^2)\mathbf{v} \times \mathbf{E}$ both for the static and the dynamic fields…The Galilean invariance of the Feigel Hypothesis… follows." [2]. It has been known since 1973 at least that these equations cannot describe fields in moving media: Lévy-Leblond and Le Bellac [3,4] have shown that these transformations do not form a group as two successive transformations do not give a similar transformation. A theory of moving media is covariant with respect to either the Lorentz transformations or the Galilean transformations of the fields, first introduced by Lévy-Leblond and Le Bellac. Indeed, they have shown that there exist two Galilean limits of the full set of Maxwell equations: the magnetic limit and the electric limit.

If one denotes $\gamma = (1 - \mathbf{v}^2/c^2)^{-1/2}$, where c is the light velocity, the relativistic transformations for the fields in vacuum between two inertial frames with relative velocity $\boldsymbol{v}$ are

$$\mathbf{E}' = \gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \frac{(1-\gamma)(\mathbf{v} \cdot \mathbf{E})\mathbf{v}}{\mathbf{v}^2}$$

$$\text{and} \quad \mathbf{B}' = \gamma(\mathbf{B} - (1/c^2)\mathbf{v} \times \mathbf{E}) + \frac{(1-\gamma)(\mathbf{v} \cdot \mathbf{B})\mathbf{v}}{\mathbf{v}^2}.$$

In vacuum, one obtains the magnetic limit by stating that $|\mathbf{v}|/c \ll 1$ and $E \ll cB$. Conversely, the electric limit is obtained by stating that $|\mathbf{v}|/c \ll 1$ and $E \gg cB$. Hence, one ends up with two sets of low-velocity formulas from the Lorentz transformations [3,4]:

$$\underline{\text{Electric Limit:}}\ \mathbf{E}' = \mathbf{E} \quad \text{and} \quad \mathbf{B}' = \mathbf{B} - (1/c^2)\mathbf{v} \times \mathbf{E}$$

$$\underline{\text{Magnetic Limit:}}\ \mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B} \quad \text{and} \quad \mathbf{B}' = \mathbf{B}.$$

van Tiggelen *et al.* [1] following Feigel [2] proposed: $\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}$ and $\mathbf{B}' = \mathbf{B} - \frac{\mathbf{v} \times \mathbf{E}}{c_L^2}$.

These transformations do not form a group for the addition that is two successive low-velocity formulas do not result in a low-velocity formula of the same form so is in contradiction with the principle of relativity. As a matter of fact, let us apply two successive changes of frame of reference:

$$\mathbf{E}_1 = \mathbf{E}_0 + \mathbf{v}_0 \times \mathbf{B}_0 \quad \text{and} \quad \mathbf{B}_1 = \mathbf{B}_0 - \frac{\mathbf{v}_0 \times \mathbf{E}_0}{c_L^2},$$

$$\begin{aligned} \mathbf{E}_2 &= \mathbf{E}_1 + \mathbf{v}_1 \times \mathbf{B}_1 \\ &= \mathbf{E}_0 + (\mathbf{v}_0 + \mathbf{v}_1) \times \mathbf{B}_0 - \frac{\mathbf{v}_1 \times \mathbf{v}_0 \times \mathbf{E}_0}{c_L^2}, \end{aligned}$$

$$\begin{aligned} \mathbf{B}_2 &= \mathbf{B}_1 - \frac{\mathbf{v}_1 \times \mathbf{E}_1}{c_L^2} \\ &= \mathbf{B}_0 - \frac{(\mathbf{v}_0 + \mathbf{v}_1) \times \mathbf{E}_0}{c_L^2} - \frac{\mathbf{v}_1 \times \mathbf{v}_0 \times \mathbf{B}_0}{c_L^2}. \end{aligned}$$

These tranformations must respect the principle of relativity; that is, the equations must keep the same form. We should have obtained the following transformations:

$$\mathbf{E}_2 = \mathbf{E}_0 + (\mathbf{v}_0 + \mathbf{v}_1) \times \mathbf{B}_0$$

$$\text{and} \quad \mathbf{B}_2 = \mathbf{B}_0 - \frac{(\mathbf{v}_0 + \mathbf{v}_1) \times \mathbf{E}_0}{c_L^2}.$$

As a conclusion, either the $\mathbf{v} \times \mathbf{B}$ term or the $\frac{\mathbf{v} \times \mathbf{E}}{c_L^2}$ term must vanish in order to respect the group additivity.

The "Galilean" equations used by van Tiggelen *et al.* [1] in bianisotropic media as well as Feigel [2] in dielectric media to prove the existence of the so-called "Feigel effect" are a mixing of these two separate Galilean transformation laws. Hence, I suggest that the effects predicted by van Tiggelen *et al.* and Feigel are not observable within the realm of Galilean Physics as they are based on wrong hypotheses. The crux of the problem relies in the use of the magnetoelectric tensor [Eq. (2) of [1]] and the authors state clearly: "We conclude that Eqs. (2) and (4) provide a Galilean-invariant description of ME effects, with the potential to be generalizable to full Lorentz invariance." The magnetoelectric tensor should be used in the context of the optics of moving bodies. Indeed, one can interpret the Feigel–van Tiggelen effect as a Casimir effect in magnetoelectric media. Yet, by definition, a Casimir-like effect features electromagnetic fluctuations that is waves so is a Lorentz-covariant phenomenon. Hence, one cannot describe the Feigel effect (seen as a Casimir effect) in a Galilean manner contrary to the conclusions of the authors: "In conclusion, we have formulated a regularized, Galilean-invariant field theory for the transfer of momentum from vacuum to magnetoelectric matter." Now, to be honest, the authors try in their reply to formulate a Lorentz-covariant theory in order to show that "zero-point momentum is allowed in a fully Lorentz-invariant model." I would leave this point to others for discussion but stand still on the impossibility to describe the Feigel–van Tiggelen effect in a Galilean way.

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