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The Maxwell-Lodge effect: significance of electromagnetic potentials in the classical theory

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Abstract. The Aharonov-Bohm effect has been the starting point of the reconsideration of the reality of the vector potential within quantum physics. We argue that the Maxwell-Lodge effect is its classical equivalent: what is the origin of the electromotive force induced in a coil surrounding a (finite) solenoid fed by an alternative current? We demonstrate theoretically, experimentally and numerically that the effect can be understood using the vector potential while it cannot using only the fields.

PACS. 03.50.De Classical electromagnetism, Maxwell equations

1 Introduction and theoretical description of the effect

According to Redhead [1], *"the gauge principle is generally regarded as the most fundamental cornerstone of modern theoretical physics. In my view, its elucidation is the most pressing problem in current philosophy of physics"*. The situation of the vector and scalar potentials (the so-called "gauge fields") is rather ambiguous in modern physics. On the one hand, quantum physics tells us that the vector potential is a "real field" following Feynman [2] thanks to the Aharonov-Bohm effect [3], the Mercereau effect [4] (*"What? Do you mean to tell me that I can tell you how much magnetic field there is inside of here by measuring currents through here and here – through wires which are entirely outside – through wires in which there is no magnetic field... In quantum mechanical interference experiments there can be situations in which classically there would be no expected influence whatever. But nevertheless there is an influence. Is it action at distance? No,* **A** *is as real as* **B***-realer, whatever that means."* said Feynman about it [5]) or the Meissner effect in superconductivity as described beautifully by Tonomura [6]. However, on the other hand, classical physics usually denies any physical meaning to the potentials. This dichotomy has now become the subject of an intense controversy among philosophers and historians of science. Moreover, the situation in quantum physics is rather strange as no less than three competing interpretations of the Aharonov-Bohm effect were proposed and no consensus was reached so far: either the Aharonov-Bohm effect is due to the vector potential (the local interpretation), or to the magnetic field (the non-local interpretation) or to the circulation of the vector potential, the so-called "holonomy" (the non-separable interpretation) [7–14].

We, physicists, will not enter into the discussion of the pros and contras arguments in favor of one of these interpretations. As a matter of fact, we claim that the multiplicity of solutions in this debate within the realm of ***quantum*** physics relies in the erroneous interpretation of the potentials in ***classical*** physics. We will present an "experimentum crucis", which, according to our point of view, is incomprehensible without the intervention of a local vector potential in classical electromagnetism: the Maxwell-Lodge effect.

Before, let us recall that, one century ago, Lorentz noticed that the electromagnetic field remains invariant under the so-called gauge transformations [15]. Hence, one must specify what is called a gauge condition, that is, a supplementary equation, which is injected in the Maxwell equations expressed in function of the electromagnetic potentials in order to suppress this indeterminacy. It is common to say that these gauge conditions are mathematical conveniences that lead to the same determination of the electromagnetic field. In this context, the choice of a specific gauge condition is motivated from the easiness in calculations compared to another one. In a certain manner, although their mathematical expressions are different, it is supposed that they are equivalent as the fields are invariant with respect to the gauge transformations.

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Furthermore, no physical meaning is ascribed to the gauge conditions as the potentials are assumed not to have one...

Despite these assertions, which are shared by a large majority of physicists, a definition for the potentials dating back to Maxwell resolves, according to our point of view, the question of the indeterminacy by giving them a physical interpretation [16–25]. For short, it is well known that the generalized momentum $\mathbf{p}$ of a particle with mass m and charge q moving at a velocity $\mathbf{v}$ in a vector potential $\mathbf{A}$ is:

$$\mathbf{p} = m\mathbf{v} + q\mathbf{A}.$$

Hence, the vector potential can be seen as the electromagnetic impulsion (per unit of charge) of the field. Following Maxwell [25]: *"The conception of such a quantity, on the changes of which, and not on its absolute magnitude, the induction currents depends, occurred to Faraday at an early stage of his researches. He observed that the secondary circuit, when at rest in an electromagnetic field, which remains of constant intensity, does show any electrical effect, whereas, if the same state of the field had been suddenly produced, there would have been a current. Again, if the primary circuit is removed from the field, or the magnetic forces abolished, there is a current of the opposite kind. He therefore recognized in the secondary circuit, when in the electromagnetic field, a "peculiar electrical condition of matter" to which he gave the name of Electrotonic State."*

As a consequence, Newton's law for the kinetic momentum becomes in classical electromagnetism Neumann's law for the electromagnetic momentum (the minus sign takes his origin in Lenz's moderation law) [16–23,25]:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \Leftrightarrow \mathbf{E} = -\frac{d\mathbf{A}}{dt}.$$

Moreover, we showed that, thanks to an analogy with fluid mechanics, the Coulomb and Lorenz gauge conditions were not equivalent because they must be interpreted as physical constraints, that is, electromagnetic continuity equations [26]. Inspired by the analogy, we were then able to demonstrate mathematically that the Coulomb gauge condition is the Galilean approximation of the Lorenz gauge condition within the so-called magnetic limit of Lévy-Leblond and Le Bellac [27]. As a matter of fact, the Galilean transformations for the potentials differ according to the two limits [27–29]. So, to "make a gauge choice" that is choosing a gauge condition is, as a consequence of our findings, not related to the fact of fixing a special couple of potentials. Gauge conditions are completely uncorrelated to the supposed indeterminacy of the potentials. Hence, we proposed to rename "gauge condition" by "constraint" [26].

We will present now a simple experiment, which, according to us, cannot be explained with Maxwell equations expressed in function of the electromagnetic field only, and which shows the physical character of a harmonic vector potential in classical physics. Indeed, let us recall that any vector can be split in three parts thanks to the so-called Stokes-Helmholtz-Hodge decomposition [30]:

$$\mathbf{A} = \mathbf{A}_{//} + \mathbf{A}_{\perp} + \mathbf{A}_h$$

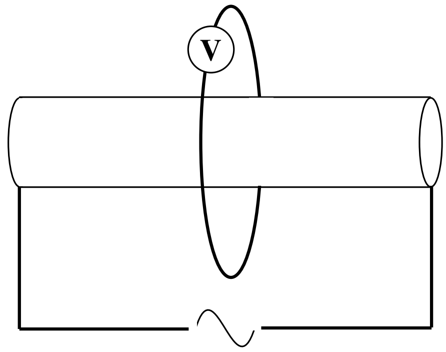

Fig. 1. The experiment of Maxwell and Lodge.

with:

$$\mathbf{A}_{//} = \nabla g,\ \ \mathbf{A}_{\perp} = \nabla \times \mathbf{R},\ \ \nabla \cdot \mathbf{A}_h = 0 \text{ and } \nabla \times \mathbf{A}_h = 0$$

where g is a scalar and $\mathbf{R}$ a vector. $\mathbf{A}_{//}$ is the so-called longitudinal part, $\mathbf{A}_{\perp}$ the transverse part and $\mathbf{A}_h$ the harmonic part. As it is well known, the Aharonov-Bohm, Mercereau and Meissner quantum effects are due to this harmonic part [30]. The Maxwell-Lodge effect demonstrates its necessity in classical physics also (as we will see). Unfortunately, the harmonic component of the vector potential was believed wrongly not to induce any effect because one can always "gauge" (remove) it by subtracting the gradient of the appropriate gauge function. However, as the space is multiply-connected, this proves to be false as the four mentioned experiments' observables are related to the circulation of the *external* vector potential (the holonomy): the phase differences in the Aharonov-Bohm and Mercereau effects, the internal magnetic flux in the Meissner effect and Maxwell-Lodge effects.

Outside an ideal solenoid of infinite length, the vector potential is precisely equal to the harmonic part (that is a gradient because $\nabla \times \mathbf{A}_h = 0$ but with $\nabla \cdot \mathbf{A}_h = 0$), as expressed by the following formula in cylindrical coordinates (we used Stokes' theorem on a closed circular path of radius r) [30]:

$$\mathbf{A} = \mathbf{A}_h = \nabla\left(\frac{\Phi\theta}{2\pi}\right) = \frac{\Phi}{2\pi r}\mathbf{e}_\theta$$

where Φ is the flux of magnetic field **inside** the solenoid or the circulation of the vector potential **outside** the solenoid. The magnetic field is null outside a perfect solenoid of infinite length in the stationary regime. Moreover, we point out forcefully that the supposed mathematical indeterminacy due to the gauge transformations is discarded by the boundary conditions which give a physical determination to the vector potential outside a solenoid: the vector potential vanishes far from its current sources.

If the current varies slowly in time, the magnetic field is still null outside the perfect solenoid but because the vector potential is not null outside the solenoid and varies with time, it creates an electric field outside the solenoid [22,31–37]:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} = -\nabla\left(\frac{d\Phi}{dt}\frac{\theta}{2\pi}\right) = -\frac{1}{2\pi r}\frac{d\Phi}{dt}\mathbf{e}_\theta.$$

If we denote the flux of the magnetic field inside the solenoid (or the circulation of the vector potential outside the solenoid) $\Phi = LI = \mu_0 n I \pi a^2$ where L is the inductance per unit length of the perfect solenoid with infinite extension, μ_0 the magnetic permeability, n the number of coil per unit length, a the coil radius and $I = I_0 \cos(\omega t)$ the current intensity (ω is the pulsation), the induced electromotive force around the entire circuit of an outside measurement coil surrounding the solenoid ($e = \oint \mathbf{E}.dI$) is expressed by:

$$e = \mu_0 n I_0 \pi a^2 \omega \sin(\omega t).$$

If we apply Maxwell equations expressed in function of the fields alone (the Heaviside-Hertz formulation) with the prescription that the magnetic field is null outside the solenoid even in this time dependent problem, we find that the electric field is harmonic outside the solenoid ($\nabla \times \mathbf{E} = 0$ and $\nabla \cdot \mathbf{E} = 0$ imply $\mathbf{E} = \mathbf{E}_h$), which is supposed to be infinite (because even in this time-dependent problem $\mathbf{B} = 0$ outside the solenoid). Of course, the mathematical resolution leads to the same expression $E_\theta = \frac{Cte}{r}$ as before when we used the potentials (the Riemann-Lorenz formulation) but the important point is that, according to Maxwell [22]: *"We have now obtained in the electrotonic intensity* [the vector potential] *the means of avoiding the consideration of the quantity of magnetic induction which passes through the circuit. Instead of this artificial method we have the natural one of considering the current with reference to quantities existing in the same space as the current itself."* This is exactly the definition of a "real field" as formulated by Feynman one century later [2]. More simply, how do the charge carriers in the external coil know that the current is varying in the solenoid?

2 Paradoxes and controversies around the Maxwell-Lodge effect

The explanation of the Maxwell-Lodge effect in term of the vector potential raises several paradoxes with respect to the appearance of an electromotive force within a circuit. Before, we have to make a historical elaboration. Indeed, let us recall that Faraday used to describe induction phenomena in a geometrical way [25]: *"the phenomena of electromagnetic force and induction in a circuit depend on the variation of the number of lines of magnetic induction which pass through the circuit"* knowing that, according to Maxwell, *"the number of these lines is expressed mathematically by the surface-integral of the magnetic induction through any surface bounded by the circuit."*

Poynting specified this point in order to visualize geometrically the vector potential [38]: *"the assumption that if we take any closed curve, the number of tubes of magnetic induction passing through it is equal to the excess of the number which have moved in over the number which have moved out through the boundary since the beginning of the formation of the field, suggests a historical mode of describing the state of the field at any moment... One can define A_x, A_y, A_z as the number of tubes of magnetic induction which would cut the axes $[(Ox, Oy, Oz)]$ per unit length if the system were to be allowed to return to its original unmagnetic condition, the tubes now moving in the opposite direction."*

This geometrical vision of induction phenomena uses the notion of lines and tubes of Force. Can these quantities, which are assistance for the visualization of fields, explain the birth of an electromotive force in the external coil?

As a matter of fact, how can the flux of magnetic induction vary through the surface delimited by the outside coil without being cut by the field tubes?

According to Poynting [38]: *"Change in the total quantity of magnetic induction passing through a closed curve should always be produced by the passage of induction tubes through the curves inwards or outwards... [However], when a part of a circuit is between the poles of an electromagnet whose magnetizing current is changing, we have no direct experimental evidence of the movement of induction in or out. But the induction tubes are closed, and to make them thread a circuit we might expect that they would have to cut through the boundary. The alternative seems to be that they should grow or diminish from within, the change in intensity being propagated along the tubes. This would be inconsistent with their closed nature* $[\nabla \cdot \mathbf{B} = 0]$, *unless the energy were instantaneously propagated along the whole length, and his further negatived by the theory of the transfer of energy, which implies that the energy flows transversely to the direction of the tubes. I shall suppose, then, that alteration in the quantity of magnetic induction through a closed curve is always produced by motion of induction tubes inwards or outwards through the bounding curve."*

In addition to the geometrical difficulty, Poynting underlines an energetic one. Indeed, the Poynting vector must be directed transversally to the direction of magnetic field's lines which is an additional indication, according to him, of the necessity of a tubes motion across the circuit so as to explain the origin of the energetic flux.

Yet, precisely, the geometry of the solenoid is such that the flux does vary, not because of tubes cutting the external coil, but, by being modify from within the solenoid (without an interaction with the coil) by a variation of the current intensity I, hence of the induction field $\mathbf{B}$ (at constant surface) which does propagate instantaneously along the tube as the solenoid "works" in the quasi-static (magnetic) limit where retardation effects are negligible. The number of tubes does *not* change but the flux does as the intensity of the magnetic induction does.

In addition, the solenoidal feature of the magnetic induction signifies that the tubes either close on themselves by forming loops or go to infinity. In the case of the perfect solenoid or the torus, the tubes present in the interior of the solenoid do not cut the external coil. For a finite solenoid, of course, the tubes close themselves in the exterior but a current variation in the solenoid does not imply their displacement or the cutting of the coil.

Concerning the energetic aspect, when the coil is an open circuit, the solenoid does not transmit energy thanks to a transverse Poynting vector as no current circulates in the coil (despite one measures an alternative tension). When the circuit is closed, there is of course a transverse energetic flux with the Poynting vector but the latter is not constructed with a hypothetic induction field coming from the solenoid (either by leaking or radiation) but with the induction field induced in the outer coil by the external electric field due to the harmonic vector potential. The balance was examined judiciously by Gough and Richards [34].

Hence, none of the arguments of Poynting are valid and the geometrical vision of Faraday and Maxwell, even if it can be fruitful in most cases does not apply to the Maxwell-Lodge effect. The reader is referred to the limpid and profound analysis of Roche on this limitation of the explanation of induction phenomena in terms of the magnetic flux lines [39].

One can establish an analogy between a solenoid and a vortex, in order to precise our discussion. Indeed, a drainage whirl in fluid mechanics can be modeled by the following equations: $\nabla \cdot \mathbf{u} = 0$ hydrodynamic continuity equation; $\mathbf{u} = \frac{\Gamma}{2\pi r}\mathbf{e}_\theta$ velocity field outside the vortex with Γ the circulation of the velocity or the flux of vorticity; $\mathbf{u} = \frac{\mathbf{w}}{2} \times r$ velocity field inside the core of the vortex with $\mathbf{w} = \nabla \times \mathbf{u}$ the vorticity and $\mathbf{a}_{cc} = \partial_t \mathbf{u}$ the resultant acceleration due to the velocity variation. For the solenoid in Electromagnetism, we have similarly: $\nabla \cdot \mathbf{A} = 0$ electromagnetic continuity equation;

$$\mathbf{A} = \frac{\Phi}{2\pi r}\mathbf{e}_\theta$$

vector potential outside the solenoid with Φ the magnetic flux or the circulation of the vector potential; $\mathbf{A} = \frac{\mathbf{B}}{2} \times r$ the vector potential within the solenoid with $\mathbf{B} = \nabla \times \mathbf{A}$ the induction field and $\mathbf{E} = -\partial_t \mathbf{A}$ the resultant electric field due to the variation of the vector potential.

To admit that the flux of magnetic field changes because of a variation of the number of magnetic tubes would imply the same thing for the vorticity associated to the vortex. However, one does not see how some vorticity coming from infinity could propagate instantaneously through the irrotational zone in order to modify the core vorticity and conversely. Besides, for a drainage vortex, one can modify the exit flow rate by sucking up which induces an increase of the vorticity from the interior of the rotational zone (this translates into an increase of the velocity hence of the acceleration in the irrotationnal zone outside the vortex). The increase of the flow rate is analogous to the increase of the electric current intensity, which induces an increase of the magnetic field inside the solenoid in the rotational zone (thus translates into an increase of the vector potential hence of electric field in the irrotational zone outside the solenoid).

Now, we will evaluate the contribution to the measured electromotive force of both the ideal vector potential as if the solenoid was infinite in length and of the leak magnetic induction due to either the finite length or the inclination of the coils. We use a spherical frame of reference (r, θ, ϕ). No electric charges are present. Hence, we are in the realm of the Galilean magnetic limit of Levy-Leblond and Le Bellac [27–29]. As a consequence, the vector potential must have a zero divergence. The fields are time-dependent but do not propagate at a finite velocity in this approximation: one speaks of instantaneous propagation. Hence, one cannot attribute the Maxwell-Lodge effect to the propagative component of the field outside the solenoid as sometimes assumed [40,41].

Let $J_\phi = I \sin\theta \delta(\cos\theta) \frac{\delta(r'-a)}{a}$ be the volume density of the current through one of the coils forming the solenoid. Following Jackson [15], the vector potential created by a coil with a static or quasi-static current I writes:

$$A_\phi = \frac{I}{sa} \int \frac{r'^2 dr' d\Omega' \sin\theta' \cos\Phi' \delta(\cos\theta'')\delta(r'-a)}{\sqrt{r^2 + r'^2 - 2rr'(\cos\theta\cos\theta'' + \sin\theta\sin\theta''\cos\phi')}}$$

with $\frac{1}{s} = \frac{\mu_0}{4\pi}$ and $\cos\theta''\delta(\cos\theta'') = 0 \Rightarrow \sin\theta''\delta(\cos\theta'') = \delta(\cos\theta'')$ such that:

$$A_\phi(r, \theta) = \frac{Ia}{s} \int_0^{2\pi} \frac{\cos\phi'\, d\phi'}{\sqrt{a^2 r^2 - 2ar\sin\theta\cos\phi}}.$$

We take into account a possible inclination of the turns forming the coil. As they wind around the coil, they always have the same angle γ. Let us denote A^i_ϕ the vector potential for an inclined turn of the coil. We can express easily its components:

$$A_r = A^i_\phi \sin\gamma\cos\theta,\ A_\theta = A^i_\phi \sin\gamma\sin\theta,\ A_\phi = A^i_\phi \cos\gamma.$$

Now, we obtain the simulated vector potential of the coil by making the sum of the vector potentials of the loops at a point M $(r,\ \theta,\ \phi)$. For symmetry reasons, it's better to switch to cylindrical coordinates $(\rho,\ \theta,\ z)$.

Let L be the height of the coil and θ_i the angle of the N_i loop with the z-axis for a point at a distance ℓ from the coil and at a height d with respect to the middle of the coil (Fig. 2). The gap between loops is ε.

We have to consider two series of turns: the first corresponding to $\theta_i < \frac{\pi}{2}$ $(N_i < N_{\rm sup})$ and the second to $\theta_i > \frac{\pi}{2}$ $(N_i > N_{\rm sup})$ with $N_{\rm sup} = {\rm INT}\left(\frac{d+L/2}{\varepsilon}\right)$ (INT means "integer part").

We can see in Figure 2 that:

$$\theta_1 = \arctan\left(\frac{\ell}{d + L/2 - N_1\varepsilon}\right) \text{ and } r = \frac{\ell}{\sin\theta_1}$$
$$\theta_2 = \arctan\left(\frac{N_2\varepsilon - (d + L/2)}{\ell}\right) + \frac{\pi}{2} \text{ and } r = \frac{\ell}{\sin\theta_2}.$$

Finally, the magnetic field $\mathbf{B}$ at the point M is obtained through the relation $\mathbf{B} = \nabla \times \mathbf{A}$:

$$B_r = \frac{\cos\gamma}{r\sin\theta}(\cos\theta A_\phi + \sin\theta\frac{\partial A_\phi}{\partial\theta})$$
$$\mathrm{B}_\theta = -\frac{\cos\gamma}{r}(A_\phi + r\frac{\partial A_\phi}{\partial r})$$
$$B_\phi = \frac{\sin\gamma}{r}\left[(A_\phi + r\frac{\partial A_\phi}{\partial r})\sin\theta + \sin\theta A_\phi - \frac{\partial A_\phi}{\partial\theta}\cos\theta\right]$$

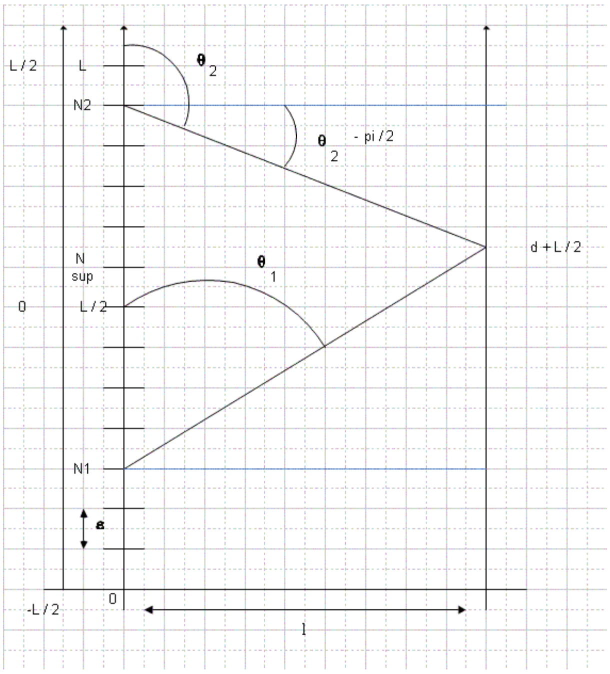

Fig. 2. Parameters for the calculation of the vector potential created by different turns at a point.

3 The experiments

3.1 Static experiments

In the following, the quantities with superscript "the", "sim" and "exp" will respectively refer to data obtained with theoretical formulae established for a solenoid of infinite length, by numerical simulations for a solenoid of limited length and by experiments.

For the experiments, we have used a coil of length $L =$ 75 cm and radius $r = 4.1$ cm made of $N = 341$ turns of 2.2 mm diameter copper wire. Its resistance is 375 mΩ and its inductance is $L = 1.08$ mH. The angle between the plan normal to the revolution axis and the plane of the loop is $\gamma = 0.027$ rad. The number of loops per unit length is $n = 454.6$ m^{-1}.

Using a F.W. Bell gauss-meter (model 4048), we measured the z component of the magnetic field B_z^{exp} versus ρ and z for a continuous current $I = 10$ A flowing in the coil (Fig. 3). It was verified that B_z^{exp} inside the coil agrees with $B_z^{the} = \mu_0 n I = 57$ gauss and that $B_z^{exp}(\rho, z)$ outside the coil agrees with $B_z^{sim}(\rho, z)$. Figure 4 gives $B_z^{sim}(\rho, 0)$ with detail ($\times$200) outside the coil: it appears that the z component of the magnetic field is not zero outside a real coil but slightly negative in this case due to the leak field; it tends to zero when ρ increases.

For what concerns the vector potential $\mathbf{A}$, one cannot measure it (as it is a momentum) but we can compare its theoretical value (for an infinite solenoid) with its simulated value (for the actual solenoid). We assume that it vanishes far from the source at infinity, which is the reference point. Note that due to the relation $\mathbf{B} = \nabla \times \mathbf{A}$, the component A_θ is the source of B_z. Inside the solenoid

$$A_\theta^{the} = \frac{\mu_0 n I \rho}{2} \text{ and outside } A_\theta^{the} = \frac{\mu_0 n I\, r^2}{2\rho},$$

these formulae agree with the simulated vector potential $A_\theta^{sim}(\rho, 0)$ represented in Figure 5.

We also evaluated by simulation the θ component of the magnetic field B_θ^{sim} due to the tilt γ of the turns and its source A_z^{sim}. It is negligible.

In conclusion of this part, a striking picture of $B_z^{sim}(\rho, z)$ is given in Figure 6 using a visualization of the intensity of the field in terms of grey levels.

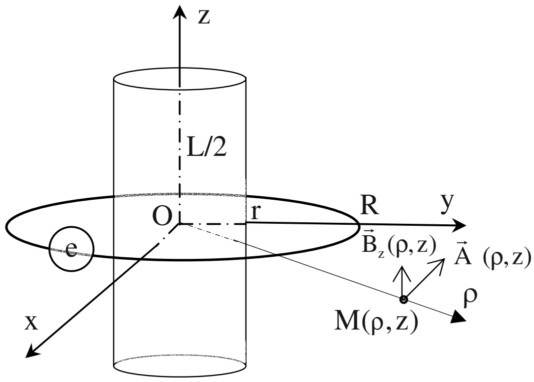

Fig. 3. The inductor and the ring used in the ML experiment.

3.2 Time-dependent experiment

Now the current flowing into the coil is sinusoidal with an intensity $I = 1$ A and a frequency $f = 1.6$ kHz. We measure and calculate the *e.m.f.* (electromotive force) e, which appears in a circular metallic ring of radius R placed in the median plane around the coil. This *e.m.f.* corresponds to the circulation of the electric field $E_\theta = -\partial_t A_\theta$. We have measured e^{exp} for rings of different diameters and calculated $e^{the} = 2\pi R \omega A^{the} = \pi \omega \mu_0 n I r^2$. We find $e^{the} = 30.3$ mV independent of the radius R of the ring while e^{exp} decreases a little for the largest rings as can be seen in the following table:

R (cm)	5	7.5	10	12.5	15
e^{exp} (mV)	28 ± 3	28 ± 3	28 ± 3	27 ± 3	26 ± 3

In order to understand this discrepancy we calculated the *e.m.f.* due to the leak field $e^{leak}(R) = 2\pi R \omega A^{leak}(R)$ with $A^{leak} = A_\theta^{sim} - A_\theta^{the}$ and compare it with $e^{sim}(R)$; the result is given in Figure 7.

Figure 7 shows clearly that the leak field plays a role in the Maxwell-Lodge effect but it does not explain the effect itself. As a matter of fact, the leak field cannot explain the total *e.m.f.* that is measured on the ring outside the coil. The leak *e.m.f.* is opposed in phase with the theoretical *e.m.f.* (constant with R) in such a way that the total *e.m.f.* decreases when the leak *e.m.f.* increases. The net result is a decrease of the total *e.m.f.* when R increases; this is exactly what we observe in e^{exp} and e^{sim}. The finite size

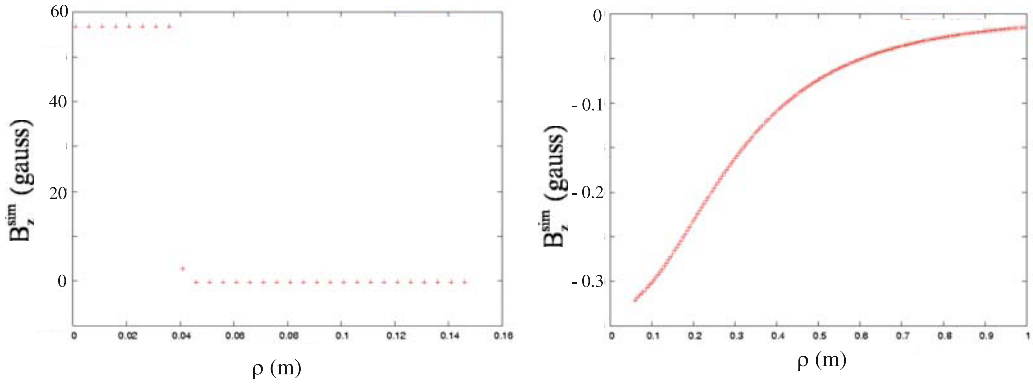

Fig. 4. (Color online) $B_z^{sim}(\rho, 0)$ component of the magnetic field simulated on the middle plane for a current of 10 A (on right, detail ×200 for the external region $\rho > 0.04$ m).

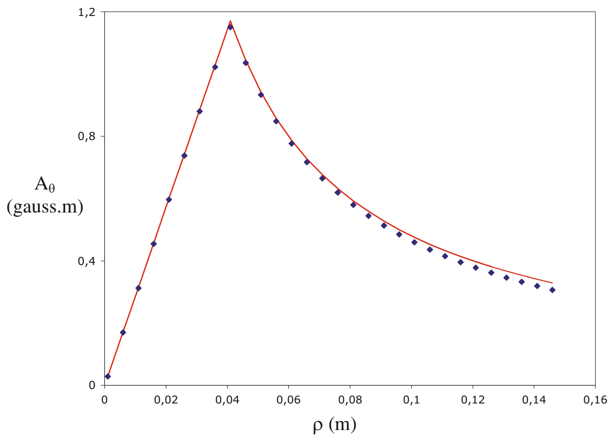

Fig. 5. (Color online) $A_\theta^{sim}(\rho, 0)$ component (dots) of the vector potential simulated on the middle plane for a current of 10 A, compared with theoretical curve $A_\theta^{the}(\rho, 0)$ (full line).

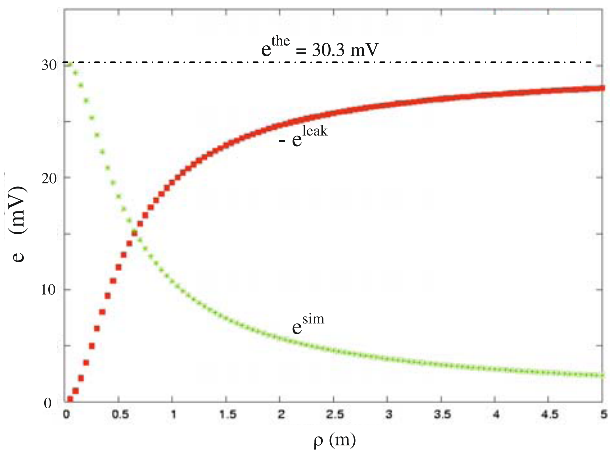

Fig. 7. (Color online) Theoretical, simulated and leak *e.m.f.* for a sinusoidal current $I = 1$ A at 1.6 kHz.

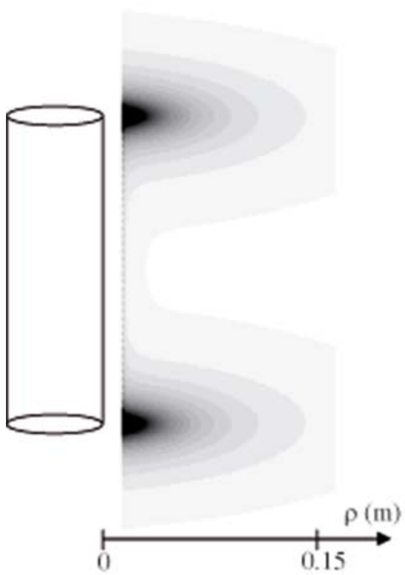

Fig. 6. $|B_z^{sim}(\rho, z)|$ for a current of 10 A. The modulus of $B_z$varies of 0.6 gauss at each grey level change.

of the coil simply makes the *e.m.f.* to decrease when going away and the tilt of the loops, that creates a B_θ component and modifies B_z in a negligible way, has no effect on it.

3.3 Check for electrostatic shielding of the coil or the ring

We made three experiments:

(i) We used a brass Faraday cage around the coil. First, we compared the electromotive force in the ring with and without the shield (Fig. 8). There is a difference: indeed, with the shield, a current sheet with a vertical z revolution axis flows in the brass cylinder and induces an internal magnetic field reverse to the field $\mathbf{B}_z$ internal to the coil. The *e.m.f.* e_2 measured at the ring is therefore reduced with respect to e_1 (*e.m.f.* without the shield).
e_2 can be calculated using the Faraday's law: $\mathbf{B}_z = \mu_0 n I \ \mathbf{e}_z$; $e_1 = \pi R^2 \omega B_z \sin \omega t$; $e_2 = e_1 \left[1 - \frac{\mu_0 \gamma R_S\, e}{2} \tan^{-1}(\omega t)\right]$ with R_S = internal radius

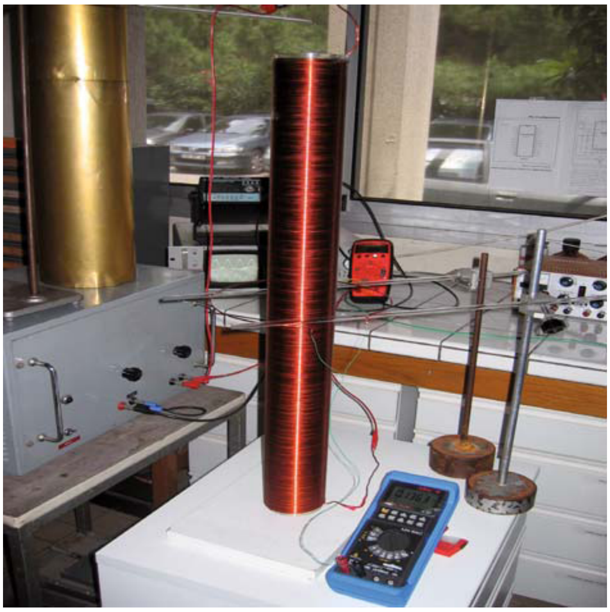

Fig. 8. (Color online) Solenoid without external shielding.

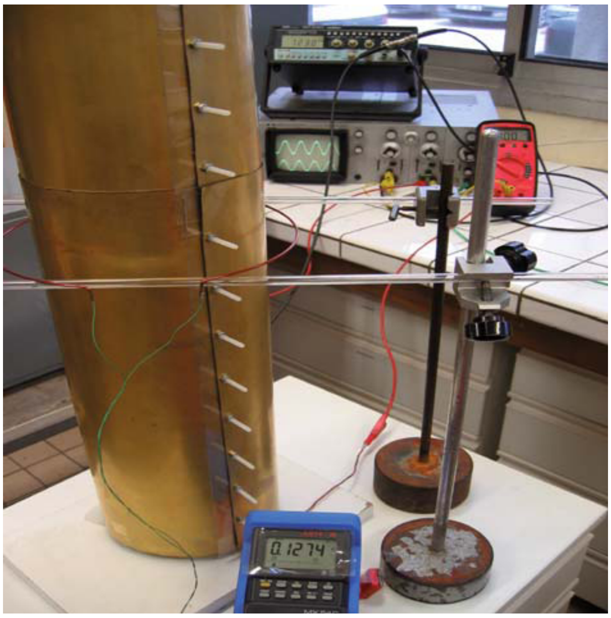

Fig. 9. (Color online) Solenoid with external shielding in brass.

of the shield, e = thickness of the shield, γ = brass conductivity.

(ii) A great part of the current induced in the shield can be suppressed by opening the brass cylinder along a line parallel to the vertical z-axis and isolating the two sides of the gap (Fig. 9). Only a capacitive effect stays, that we can evidence because of its increases with frequency. In these conditions, the emf e_2 measured at the ring is very near to e_1:

$$1 \text{ kHz: } e_2 = 0.992 e_1,$$

$$2 \text{ kHz: } e_2 = 0.965 e_1,$$

$$4 \text{ kHz: } e_2 = 0.954 e_1,$$

$$7 \text{ kHz: } e_2 = 0.928 e_1,$$

$$9 \text{ kHz: } e_2 = 0.920 e_1.$$

Note that there is no difference if the shield is grounded.

(iii) We can shield the ring instead of the coil. For this, we used a coaxial cable instead of the usual ring (the measures are made at the internal conductor and the shield of the cable is grounded or floating). We observe a small effect that we also attribute to capacitive effects.

In conclusion, these complementary experiments do not show any effect of an electrostatic shielding on the vector potential outside of the solenoid.

4 Conclusions

We proposed a complete description of the Maxwell-Lodge effect which conclusion is the necessity to use the vector potential to interpret it. The electromotive force induced by a changing current in a solenoid through an outer coil is due to the vector potential outside the solenoid. The effect has nothing to do with either the propagative component of the magnetic field, a possible inclination of the coils forming the solenoid or the leak magnetic field due to the finite length of the solenoid. Then, we propose the vector potential, usually considered as a "mathematical tool", to become a "real field" in the sense introduced by Richard Feynman [2]. According to the Nobel Prize C.N. Yang [21]: *"throughout most of 20th century the Heaviside-Hertz form of Maxwell's equations were taught to college students all over the world. The reason is quite obvious: the Heaviside-Hertz form is simpler, and exhibits an appealing near symmetry between* **E** *and* **H**. *With the widespread use of this vector-potential-less version of Maxwell's equations, there arouse what amounted to a dogma: that the electromagnetic field resides in* **E** *and* **H**. *Where both of them vanish, there cannot be any electromagnetic effects on a charged particle. This dogma explains why when the Aharonov-Bohm article was published it met with general disbelief...* **E** *and* **H** *together do not completely describe the electromagnetic field, and... the vector potential cannot be totally eliminated in quantum mechanics... the field strengths underdescribe electromagnetism."*

The question that we raise after many others is the following: why an electric field is created outside a perfect solenoid when current varies in it with time? Our answer is that the vector potential is not null outside a perfect solenoid contrary to the magnetic field so is responsible for the outside electric field.

However, solenoids are not perfect except if we use a toroidal superconductor solenoid like Tonomura et al. in order to demonstrate the existence of the Aharonov-Bohm effect hence the physical role of the vector potential in quantum physics. Here, we are interested with classical physics and with a setup that any physicists can build easily. Now the question becomes: why an electric field is created outside a real solenoid when current varies in it with time?

Let us resume our main claims :

- electromagnetic waves cannot be responsible for the Maxwell-Lodge: indeed the wavelength are much longer that the size of the setup;
- imperfections like the fact that the solenoid is an helix and not a superposition of horizontal coil were dismissed due to the smallness of the effect;
- the important point: a magnetic field does exist outside a real solenoid because the solenoid features extremities from where the inner magnetic field leaks outside. In addition, due to to the fact that the magnetic field is divergenceless, the magnetic field lines from both ends of the solenoid must reconnect. But, our major result is to have shown that the additional electric field due to the leaking magnetic field is in opposition to the electric field due to the vector potential of the associated real solenoid. So, the vector potential is truly the main cause of the outside electromotive force acting on the coil. The only other way to explain the Maxwell-Lodge effect is through a non-local influence of the inner magnetic field outside the solenoid. We prefer to use a local formulation of classical electromagnetism based on the local vector potential.

As possible extensions of our work, refinements of the theoretical model would be interesting in order to explain the small discrepancies between our experimental electromotive forces and the theoretical ones derived from our simple analysis. Indeed, the aspect ratio between the radius and the length of a finite solenoid is an important quantity as discussed in the context of the AB effect by Babiker and Loudon [42] despite the fact that the authors attributed (wrongly) the AB effect to the transverse part of the vector potential. Moreover, the real geometry of the solenoid can be handled easily thanks to the computations of the resultant helical vector potential [43,44].

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